# Seismic Response Analysis of Different Buildings using Time- Invariant and Time- Variant Damping Coefficients

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## Abstract

Seismic signals are characterized by strong excitations, short durations, non-linearity and non-stationarity having both the amplitude and frequency content vary as a function of the recorded time. Various classical detection and estimation techniques, like the time-frequency representations and the Fourier-based techniques have been used to analyze such signals, but these techniques have variety of limitations and they fail to correctly estimate the concerned signals. The Damped-Amplitude and Polynomial-Frequency Model has been introduced to help in adapting to the seismic signals where the amplitude is damped. This model is based on approximating the frequency by low-order polynomials and the amplitude by damped exponentials. Its amplitude in turn is characterized by a damping coefficient; which was firstly assumed to be time-invariant. However the results of the studied signals showed rapid amplitude fluctuations and frequency content variations of each component that could be justified by the fact that the dynamic response of the structure is highly sensitive to the dynamic characteristics of the ground motion. Accordingly and to be more adapted with the physical model of the building motion that is characterized by damped exponential functions, the damping coefficient was then assumed to be time-variant leading to the foundation of a new model that keeps the same approximation for the frequency like the aforementioned model, and changes that of the amplitude by approximating its damping-coefficient by low-order polynomials. This model was then named Polynomial Damping Function Model. Results on different seismic signals show that the time-variant assumption is more efficient than the time-invariant one.

## 1. Introduction

Seismic signals are non-stationary; having both the amplitude and frequency content vary considerably as a function of recording time. Consequently, for a more reliable representation of such signals, both the amplitude and frequency variations of the recorded time histories should be accounted for. Recently, a large number of methods have been proposed in this context. For example, Reine et al. (2009) [(1)] compared time-frequency and time-scale methods in seismic data processing and interpretation, the S-transform method was also applied in seismic data analysis by Pinnegar and Mansinha

(2003) [2], and the reassignment method was proposed for the small frequency variations in civil engineering structures under weak and strong motions by Michel and Gueguen (2009) [3]. In the context of the signals under study in this paper, these techniques present certain drawbacks. The analysis of [1, 2, and 3] is based on the time-frequency representation which is limited in time-frequency resolution by the choice of a window length. Particular drawback in [3] is that the reassignment of the energy yields wrong amplitude estimations. Accordingly, and in order to approach the dynamic evolution of the modulations, a high time resolution is required, and to have better performance in multi-component case, a high frequency resolution is needed.

Generally, to get away from the constraints and drawbacks of such non-parametric classical methods, and to correctly reconstruct the signals under study, a model should be set. This model should be as general as possible to be applicable in different domains.

In previous papers, we already proposed a polynomial frequency model with polynomial amplitude in [4, 5, 6], or with damped amplitude in [7, 8]. In this paper, a new model named polynomial damping function model is proposed for the amplitude modulation. This model is of great interest since it is still able to deal with multi-component signals of very short durations, and with the rapid non-linear amplitude and frequency variations of each component of a signal. Moreover, compared to the damped amplitude model in [7, 8], the damping coefficient of its amplitude is assumed to be time variant, so that it is less constrained and better convenient for damping factor estimation of seismic signals.

The model proposed in this paper needs a parameter estimation method, and a procedure to solve the non-linearity problem of the likelihood function similar to those of [7, 8], which are the Maximum Likelihood Estimation (MLE) and the Adaptive Simulated Annealing (ASA) techniques respectively. The objective of this paper is then to apply the method proposed herein to estimate the amplitude and frequency modulations (AM/FM) of seismic vibration signals of buildings, recorded by different accelerometers placed on the top of the buildings.

The remainder of the paper is structured as follows. In section 2, the physical model for the seismic vibration signals of interest is formulated. The signal model for the AM/FM estimations and the parameter estimation method are introduced in section 3. The obtained results over the analysis of simulated signals computed from the physical model and real-world seismic data are discussed in section 4. Finally, the paper ends with final conclusions in section 5.

## 2. Physical model of the building motion

In [9] the dynamic behavior of a building is deduced from the matching of the building with a continuous Timoshenko beam. In this model, the mass of each storey of the building is considered to be mostly concentrated at its floor, and a lumped mass modeling is assumed for this structure. Therefore, the Duhamel integral gives the elastic motion of the building at each floor $U_{fl}(t)$ by only knowing the mass of each floor, the modal parameters (namely the mode shapes , frequencies  and damping ratios), and the ground motion $P(t)$.

The mode shape of the $fl^{th}$ floor $\phi_k\left[fl\right]$ depends on the number of the floor $fl$ and it participates as the weighting factor to generate the final response of the structure, as will be shown later in this section. The impulse response of a single degree of freedom system for a $k$ mode is

$$h_k(t) = \frac{-1}{\omega_{Dk}} e^{-\xi_k \omega_k t} \sin\left(\omega_{Dk} t\right), \forall k \in \left[1, N_{\mathrm{mod}e}\right], \tag{1}$$

with $\omega_{Dk} = 2\pi f_k \sqrt{1-\xi_k^2}$ the pseudo-pulsation of mode $k$. This system being excited by the ground solicitations, the time-varying response of the mode $k$ is obtained from the convolution with the ground solicitation $P(t)$ and is given by the Duhamel integral

$$y_k(t) = p_k \int_0^t P(\tau).h_k(t-\tau)d\tau = p_k\ P(t) * \frac{-1}{\omega_{Dk}} e^{-\xi_k \omega_k t} \sin(\omega_{Dk} t), \tag{2}$$

with $p_k$ the participation factor that is defined as $p_k = \dfrac{\sum_{fl=1}^{N_{fl}} \phi_k\left[fl\right]}{\sum_{fl=1}^{N_{fl}} \phi_k^2\left[fl\right]}$.

In fact, a building is more complex than a single degree of freedom system, so that the motion of the building is then modelized as a linear superposition of the motion of all the $N_{\mathrm{mod}e}$ modes. Therefore, the dynamic response for a given floor  with $fl \in \left[1, N_{fl}\right]$ is obtained by superimposing the $N_{\mathrm{mod}e}$ modal responses $y_k(t)$ obtained in (2)

$$U_{fl}\ (t) = \sum_{k=1}^{N_{\mathrm{mod}e}} y_k(t) \phi_k\left[fl\right]. \tag{3}$$

From this point, it is worthwhile mentioning that the non-stationary dynamic response of the structure is highly sensitive to the dynamic characteristics of the ground motions. In the next two sections, two cases will be studied, the ambient ground excitations and the seismic ground excitations.

Accordingly, as an initial step to clarify this physical model, and to introduce the next two sections, let's consider one example with the continuous physical model of a commercial building, located in Sherman-Oaks, California, assuming that the mass of each floor is constant, and the number of floors being 13 (see Fig. 1 left). From the spectrogram and the Fourier spectrum of the data recorded at the top of this building when excited by the real-world earthquake of Northridge January 17, 1994 ($M_L = 6.4$) (see Fig. 1 middle and right respectively), the number of modes $N_{\mathrm{mod}e}$ could be clearly seen as 3 located at 0.4, 1.2 and 2.2 Hz respectively. Thus, knowledge of the first two modal frequencies $f_1$=0.4 Hz and $f_2$=1.2 Hz, necessary in the Timoshenko beam model [10], allows evaluating the third modal frequency at 2.2 Hz which is perfectly matched with what is presented by the spectrogram and the Fourier spectrum of this signal.

For each mode $k$, the damping ratio $\xi_k$ is assumed to be fixed at 0.05, accordingly the pseudo pulsations $\omega_{Dk}$ of this building will be $\omega_{D1} = 2.504$ .rad/s, $\omega_{D2} = 2.712$ rad/s, and $\omega_{D3} = 4.972$ rad/s.

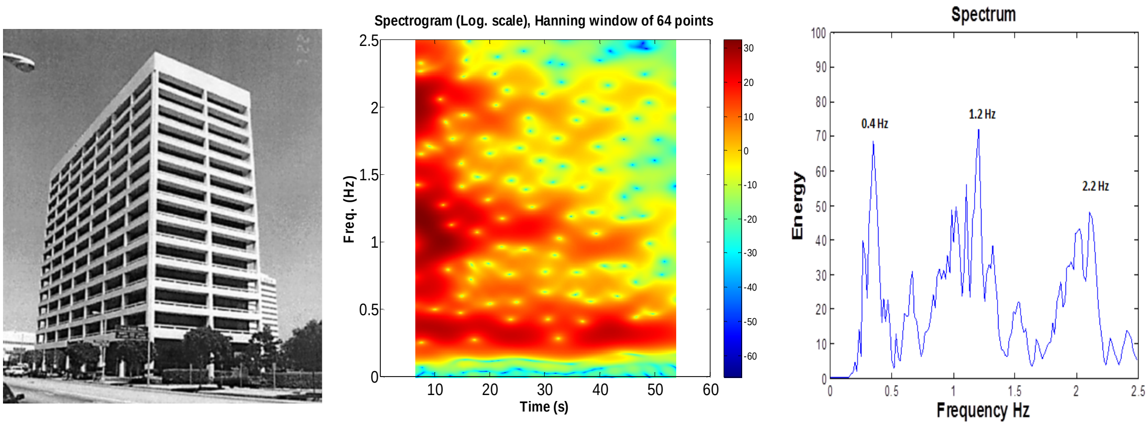



**Figure 1: The 13-storey commercial building located in Sherman-Oaks, California (left), the spectrogram calculated with Hann window of 12.8 s (64 points) (middle), and the Fourier spectrum (right), of the data recorded at the top of the Sherman-Oaks building when excited by the real-world earthquake of Northridge January 17, 1994 ( $M_L$ = *6.4* )**

Figs. 2 (left and right) present respectively the mode shapes of the 13th floors and the impulse response of the 3 modes of the Sherman-Oaks building.

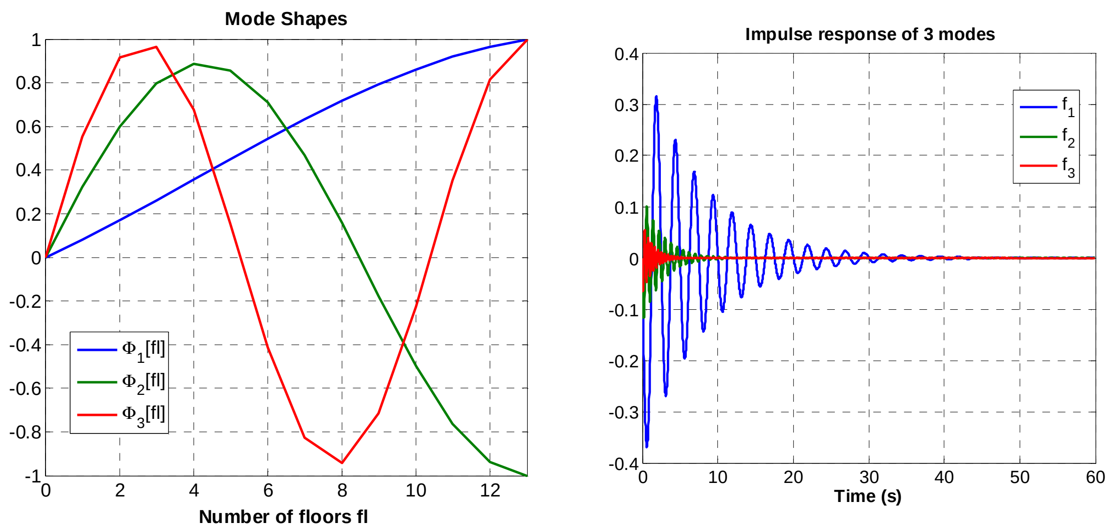



**Figure 2: The mode shapes of the 13th floor (left) and the impulse response of the 3 modes (right) of the Sherman-Oaks building**

### *2.1 Under Ambient ground excitations*

Ambient vibrations in buildings are produced by the wind (low frequencies < 1Hz), internal sources (machinery, lift at high frequencies) and seismic noise (broadband). The seismic noise being prevailing in case of ambient ground excitations, the input in model (2) is then assumed to be a stationary white noise with zero mean and constant variance.

For the example of Sherman-Oaks building of Fig. 1, when excited by such a ground excitations, the result of Eq. (2) for simulating modal responses $y_k(t)$ for the 3 modes, and the result of Eq. (3) for simulating the full dynamic response will be as shown in Fig (3).

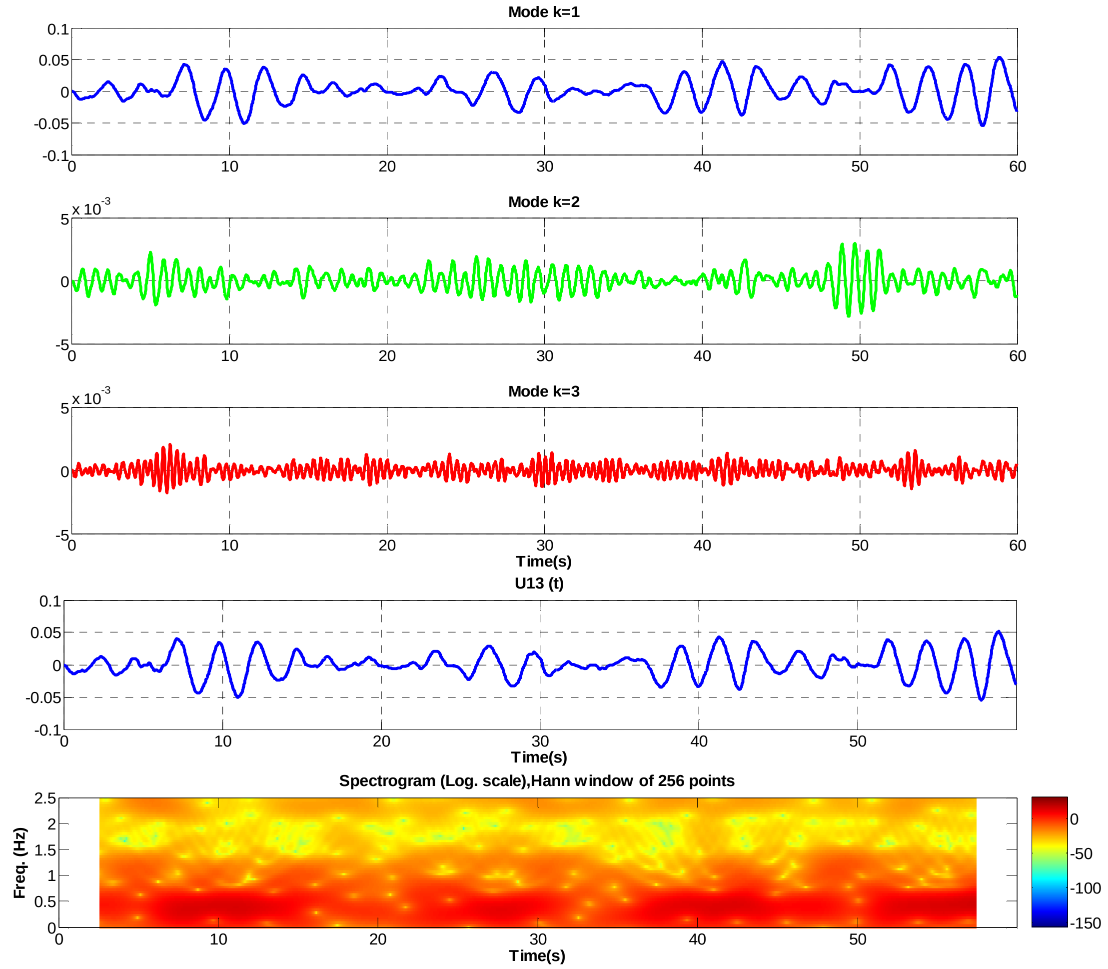



**Figure 3: Dynamic response model at the top of Sherman-Oaks building when excited by white Gaussian ground excitations: modal responses $y_k(t)$ of the 3 modes, mode k=1, 2 & 3 (1st line top, 2nd line, 3rd line respectively), the dynamic response of the 13th floor $U_{fl}(t)$ (4th line), and its spectrogram (Hann window of 51.2 s-256 points) (5th line)**

## 2.2 *Under Seismic ground excitations*

Seismic ground excitations are more difficult to be modeled, and their spectral shape is no longer white. In this paper, these seismic excitations will be a real-world ones recorded at the ground level of the Sherman-Oaks building of Fig. 1 during the earthquake of Northridge January 17, 1994 ($M_L = 6.4$). For the most energetic part of this signal between 2 and 12 s, as shown by the spectrogram in Fig. 4 left, the spectral shape is no longer white. The log-log presentation of the spectrum of this energetic part, (see Fig. 4 right), corresponds actually to the Brune model [11] with a flat frequency band between $f_c = 0.6$ Hz and $f_{\max} = 5$ Hz.

Such an input in Eq. (2) provides the modal response $y_k(t)$ of each mode and the full dynamic response of the Sherman-Oaks building to the earthquake considered (see Fig. 5).

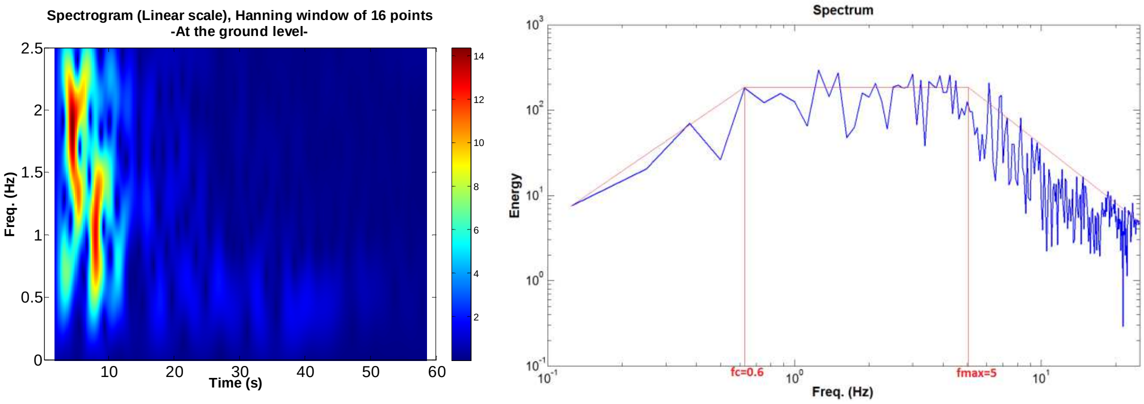



**Figure 4: The spectrogram calculated with Hanning window of 3.2 s (16 points) (left), and the log-log presentation of the Fourier spectrum around the most energetic part of the seismic data [2-12s] corresponds to Brune Model (right) of the seismic data recorded at the ground level of the Sherman-Oaks building**

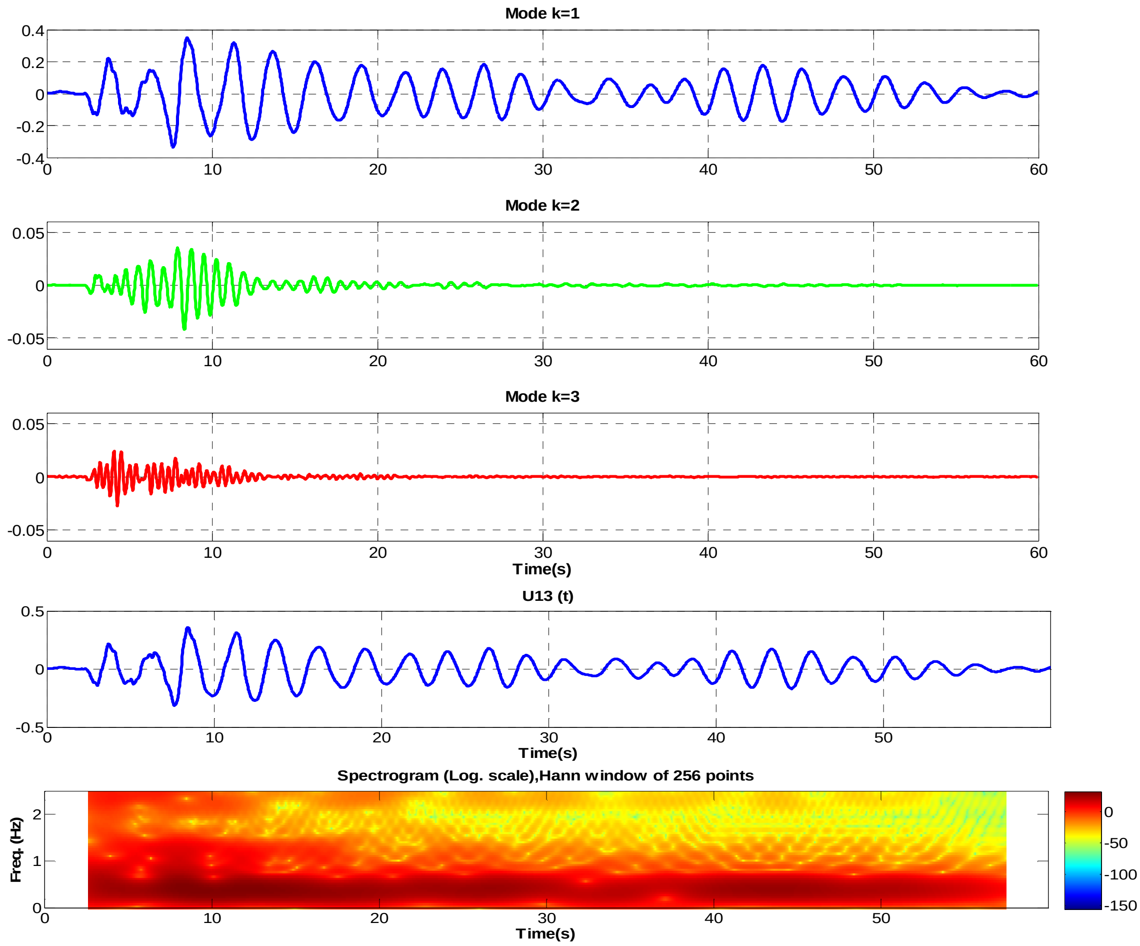



**Figure 5: Dynamic response model at the top of Sherman-Oaks building when excited by the real-world earthquake of Northridge Jan. 17, 1994 ($M_L = 6.4$): modal responses $y_k(t)$ of the 3 modes, mode k=1, 2 & 3 (1st line top, 2nd line, 3rd line respectively), the dynamic response of the 13th floor $U_{fl}(t)$ (4th line), and its spectrogram (Hann window of 51.2 s-256 points) (5th line)**

Generally, different types of excitations will affect the building response differently. Whereas for the ambient ground excitation, all the modal responses in Fig. 3 (1st, 2nd and 3rd lines) are affected in the same way by ground motions during the whole recorded time series, where for seismic ground excitations case (see Fig. 5), the response increases in presence of the seism between 2-12s, then return to the amplitude variations due to the ambient ground excitations.

Comparing the two spectrograms, (Fig. 3 (5th line) and Fig. 5 (5th line)) show that the mode at 0.4 Hz is more energetic than the other two at 1.2 and 2.2 Hz. These last two modes have also been modified by the earthquakes. Consider the energy of the mode $k$ $E_k$ calculating in the most energetic part between 2-8s as follows

$$E_k = \sum_{t=2s}^{8s} \left( y_k(t) \right)^2 \qquad \forall k \in [1, N_{\mathrm{mod}e}] \tag{4}$$

where $y_k(t)$ is the modal response of the mode $k$. Table 1 shows an energy calculation of all the modes for both ambient and seismic excitations during the seismic period between 2-8s.

**Table 1: Energy calculation for each mode of the ambient and seismic excitations**

| | $E =$ Energy of the Ambient excitations | $E' =$ Energy of the Seismic excitations | $\Delta E = \dfrac{E}{E'}$ |
|---|---|---|---|
| Mode 1 at 0.4 Hz | 5.24 | 0.1141 | 45.9 |
| Mode 2 at 1.2 Hz | 0.0468 | $1.044 * 10^{-4}$ | 448.2 |
| Mode 3 at 2.2 Hz | 0.0278 | $9.351 * 10^{-5}$ | 297.3262 |

## 3. Signal models in view of AM/FM estimations

In this paper, we intend to estimate the AM/FM of these seismic signals. A signal model $y[n]$ is defined as a discrete time signal with time varying amplitude and frequency, having short time duration, and composed of a multi-component deterministic process $s[n]$ embedded in an additive white Gaussian noise $e[n]$ with zero mean and variance $\sigma^2$.

$$y[n] = s[n] + e[n] \quad \text{where} \quad s[n] = \sum_{i=1}^{K} A_i[n] e^{j\Phi_i[n]}. \tag{5}$$

The time reference is set to the center of the window as $-\frac{N}{2} \le n \le \frac{N}{2}$, $K$ is the number of components, $j$ is the complex number verifying $j^2 = -1$, $A_i[n]$ is the time-varying amplitude, and $\Phi_i[n]$ is the phase of the $i^{th}$ component, obtained by numerical integration of the Instantaneous Frequency IF, and centered in the middle of the observation window in order to minimize the estimation error [(4)], thus making $\varphi_{i,0} = \Phi_i[0]$,

$$\Phi_i[n] = \varphi_{i,0} + 2\pi \left( \sum_{k=-N/2}^{n} F_i[k] - \sum_{k=-N/2}^{0} F_i[k] \right) \tag{6}$$

To assure the uniqueness of the model, the Instantaneous Amplitude (IA) should be strictly positive $A_i[n] > 0$, the phase $\Phi_i[n]$ should not contain any discontinuities, and the IF should respect Shannon's theorem $0 < F_i[n] < F_s/2$ where $F_s$ is the sampling frequency [5, 6].

### *3.1 Damped amplitude and polynomial frequency model*

For the frequency modulation model, like in [7, 8], the IF is approximated by low order polynomials, in order to track the strong local variations,

$$F_i[n] = \sum_{m=0}^{M_f} f_{m,i}\, g_m[n], \tag{7}$$

where $M_f$ is the approximation order of IF, $g_m[n]$ is an orthonormal polynomial of order m, and $f_{m,i}$ is the frequency parameter.

In this paper, the amplitude modulations are approximated by two types of damped functions, one with time-invariant damping coefficient like in [7, 8] and another, which is the proposition of this paper, with time-variant polynomial damping coefficient. Table 2 presents these two studied signal models and their characteristics.

#### *3.1.1 Time-invariant damping coefficient*

In this section, the damping coefficient of the structure is assumed to be time-invariant, so that

$$A_i[n] = \beta_i e^{-\alpha_i n}, \tag{8}$$

where $\beta_i$ and $\alpha_i$ are the initial amplitude and the damping coefficient respectively, characterizing the amplitude of the $i^{th}$ component of this model. With regard to the real-world data, $\beta_i$ is always constrained to be positive [7, 8].

In summary, the P parameters to be estimated are $\theta_i = \left[\beta_i, \alpha_i, \varphi_{i,0}, f_{i,0}, \ldots, f_{i,M_f}\right]$ with $P = M_f + 4$.

#### *3.1.2 Time-variant polynomial damping coefficient*

The real seismic signals are characterized by rapid amplitude fluctuations and frequency variations of the energy contents of each of their components, from this point the damping coefficient was assumed to be time-variant and thus a new model known as "Polynomial Damping Function Model" is proposed to approximate the amplitude fluctuations. This model is more adapted to the civil engineering buildings that are characterized by the exponentially damping functions, and thus it is in well accordance with the physical model presented in section 2.

For this model, the amplitude is defined as follows

$$A_i[n] = \beta_i e^{-\left(\sum_{k=-N/2}^{n} \alpha_i[k] - \sum_{k=-N/2}^{0} \alpha_i[k]\right)} \quad \text{where} \quad \alpha_i[n] = \sum_{m=0}^{M_\alpha} \rho_{m,i}\, g_m[n], \tag{9}$$

with $\beta_i$ being the initial amplitude, $M_\alpha$ is the approximation order of the damping coefficient, $g_m[n]$ is an orthonormal polynomial of order m, and $\rho_{m,i}$ are the amplitude parameters.

Accordingly, the P parameters to be estimated are $\theta_i = \left[ \beta_i, \rho_{i,0}, \ldots, \rho_{i,M_\alpha}, \varphi_{i,0}, f_{i,0}, \ldots, f_{i,M_f} \right]$ with $P = M_f + M_\alpha + 4$.

**Table 2: The two studied signal models**

| | Time-Invariant Damping Coefficient - Damped Amp.& Polynomial Freq. Model - | Time-Variant Damping Coefficient -Polynomial Damping Function Model- |
|---|---|---|
| (IA) | $A_i[n] = \beta_i e^{-\alpha_i n}$ | $A_i[n] = \beta_i e^{-\left(\sum_{k=-N/2}^{n} \alpha_i[k] - \sum_{k=-N/2}^{0} \alpha_i[k]\right)}$ |
| (IF) | $F_i[n] = \sum_{m=0}^{M_f} f_{m,i}\, g_m[n]$ | $F_i[n] = \sum_{m=0}^{M_f} f_{m,i}\, g_m[n]$ |
| $(\alpha_i)$ | $\alpha_i$ constant | $\alpha_i[n] = \sum_{m=0}^{M_\alpha} \rho_{m,i}\, g_m[n]$ |
| $(P)$ | $M_f + 4$ | $M_f + M_\alpha + 4$ |
| $(\theta_i)$ | $\theta_i = \left[ \beta_i, \alpha_i, \varphi_{i,0}, f_{i,0}, \ldots, f_{i,M_f} \right]$ | $\theta_i = \left[ \beta_i, \rho_{i,0}, \ldots, \rho_{i,M_\alpha}, \varphi_{i,0}, f_{i,0}, \ldots, f_{i,M_f} \right]$ |

### *3.2 Parameter estimation method*

A maximum-likelihood approach as in [7, 8] is used to estimate the model parameter vector $\theta_i$. As the noise is assumed to be a white Gaussian process; the MLE is equivalent to minimization of a least Square approach,

$$\hat{\theta} = \underset{\theta \in R^{K \times P}}{\arg\min}\; l_{LS}(\theta), \text{ with } l_{LS}(\theta) = \sum_{n=-N/2}^{N/2} \left| y[n] - \hat{s}[n] \right|^2, \tag{10}$$

with $y[n]$ the noisy observation, and $\hat{s}[n]$ the signal model computed by substituting $A_i[n], F_i[n]$ and $\Phi_i[n]$ in $s[n]$.

Analytical solutions are not applicable on (10) due to its non-linearity, the same stochastic optimization technique as in [7, 8] is then applied. This technique speeds up the convergence process to the global optima and induces a gain in the computing time. The detailed steps of this technique can be found in [13].

## 4. Applications on seismic signals

Firstly in this section, the aforementioned two signal models of section 3 will be applied over the real-world seismic data recorded at the top of three different buildings:

- 1st building: The 7-storey hotel building located in Van Nuys, California, Fig. 6 (left), which was severely damaged by the Northridge earthquake of January17, 1994, ($M_L = 6.4$). The signal is of 60 seconds (3000 samples) length in total, sampled at 50Hz and decimated at 5Hz.
- 2nd building: The 13-storey governmental office San José building located in the Bay area of California, San Francisco, Fig. 6 (right) which was severely damaged as well by the earthquake of Loma Prieta of October 17, 1989 ($M_L = 6.9$). The signal is of 120 seconds (6000 samples) length in total, sampled at 50Hz and decimated at 10Hz.

- 3rd building: The 13-storey commercial building, located in Sherman-Oaks, California, already presented in Sec. 2 and Fig. 1. This building has been damaged by the earthquake of Northridge January 17, 1994 ($M_L = 6.4$), The signal is of 60 seconds (3000 samples) length in total, sampled at 50Hz and decimated at 10Hz.

The two signal models considered will be evaluated over these real-world seismic data and compared all together in the aim of highlighting the interest of the model proposed in this paper when dealing with such data. Then at the end of this section, the proposed model will be applied over the simulated seismic signals of section 2.

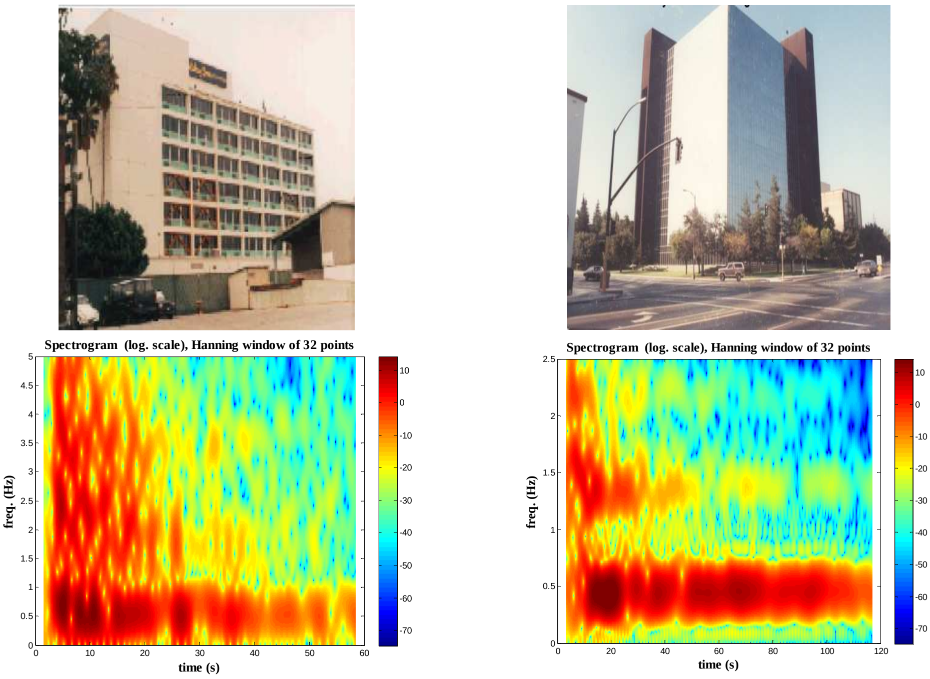



**Figure 6: 7-storey hotel building located in Van-Nuys, California (top left), 13-storey San-José building located in the Bay area, California (top right), and the spectrograms of the real-world seismic data recorded at the top of the Van-Nuys and San José buildings calculated with Hann window of 3.2 s (32 points) and 6.4 s (32 points) (bottom left and bottom right respectively)**

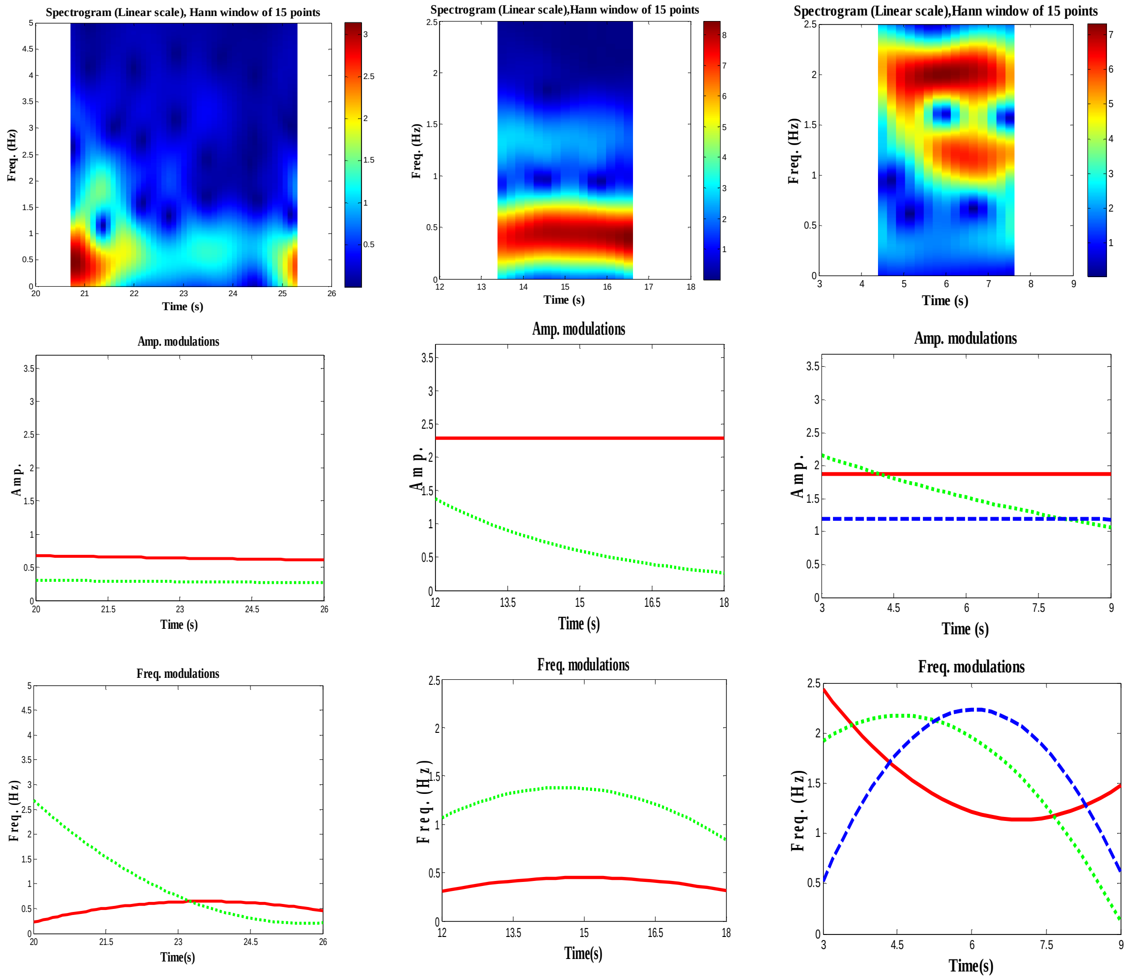
### *4.1 Analysis of real-world seismic data*

For each of the three real signals presented in the previous section, an appropriate segment of 6 s length is studied. For the case of the time-invariant damping coefficient, the frequency is estimated at the 2nd order. While for the case of time-variant damping coefficient, the polynomial damping function is estimated at the 3rd order.

#### *4.1.1 Time-invariant and time-variant damping coefficient*

In [7, 8], the initial amplitude $\beta_i$ and the damping coefficient $\alpha_i$ are constrained to be strictly positive (Sec. 4.1).

Fig. 7 presents the results of the damped amplitude and polynomial frequency model, with $\alpha_i$ constrained to be strictly positive, when studied over the three real-world seismic data of the buildings presented in section 4. This model has shown its efficiency when applied over ambient vibration signals [7, 8]. Our aim in this paper is mainly to test this model over real-world seismic data, which are characterized by more intense vibrations.

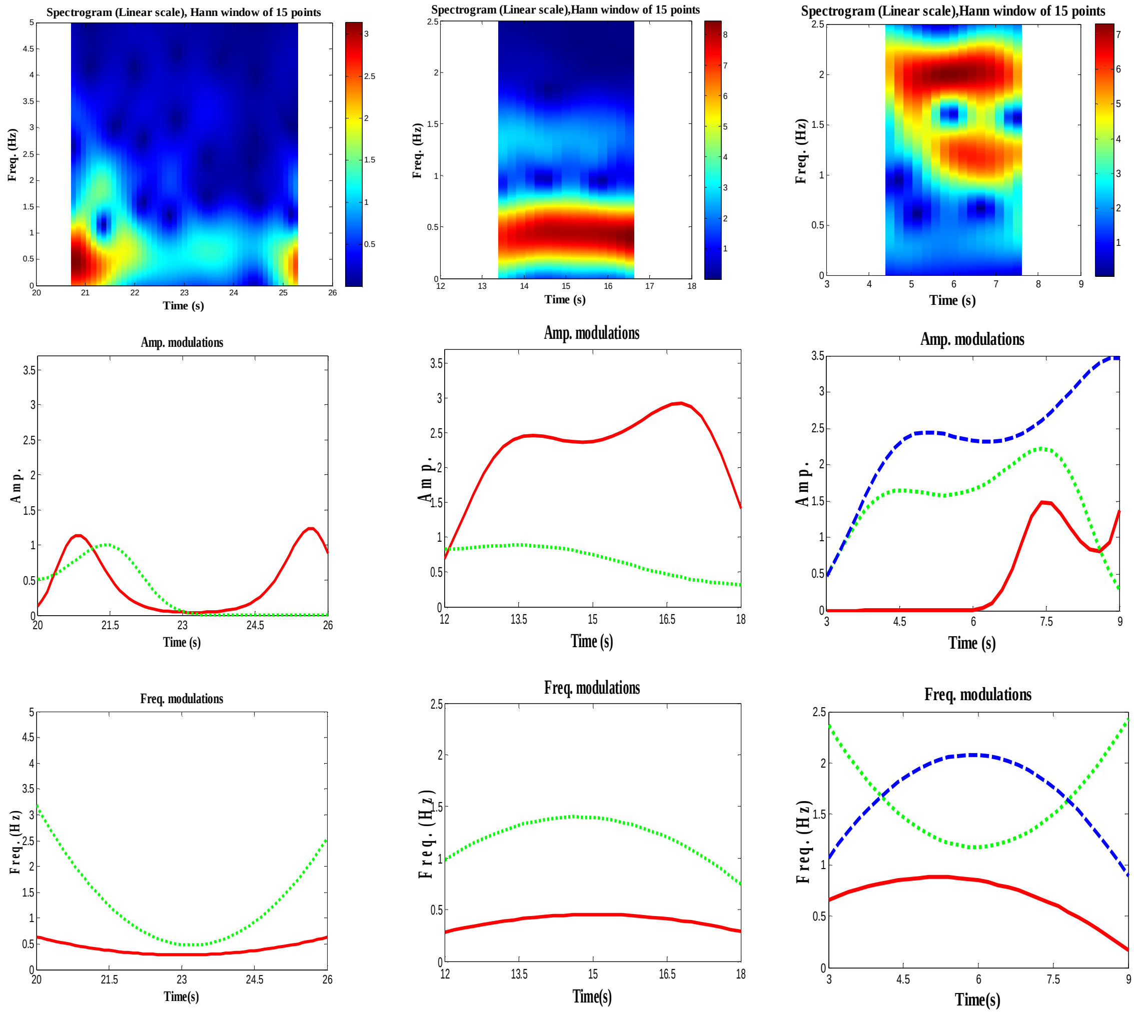



Normalized Error = 0.2047 Normalized Error = 0.0682 Normalized Error = 0.1215

**Figure 7: Analysis of the real-world seismic data recorded at the top of Van-Nuys hotel (1st column left), San-José building (2nd column middle) and Sherman-Oaks building (3rd column right) when using the damped amplitude and polynomial frequency model ($\alpha_i$ constant): The spectrograms calculated with Hann window of 15 points (1st line top), the estimated amplitude modulations (2nd line middle), and the estimated frequency modulations (3rd line bottom)**

The results reported in Fig. 7 proved that this model is no longer reliable when dealing with seismic data. The amplitude modulations of the three chosen studied signals are not correlated with the spectrogram, which indicates that the local variations of the concerned signals weren't tracked correctly using this model. From this point, and in the aim of improving the performance of this model, $\alpha_i$ was then assumed to be without constraints. The results of this hypothesis, studied but not presented here, didn't improve the performance at all, neither the normalized errors were minimized, nor the amplitude variations were in agreement with the spectrogram.

Based on these conclusions and contrary to the damped amplitude and polynomial frequency model that is concentrated on the time-invariant damping-coefficient, in this paper, a new model namely the polynomial damping function model that accepts $\alpha_i$ to be time-variant is proposed (c.f. section 3.1.2).




Normalized Error = 0.1997 Normalized Error = 0.0121 Normalized Error = 0.0963

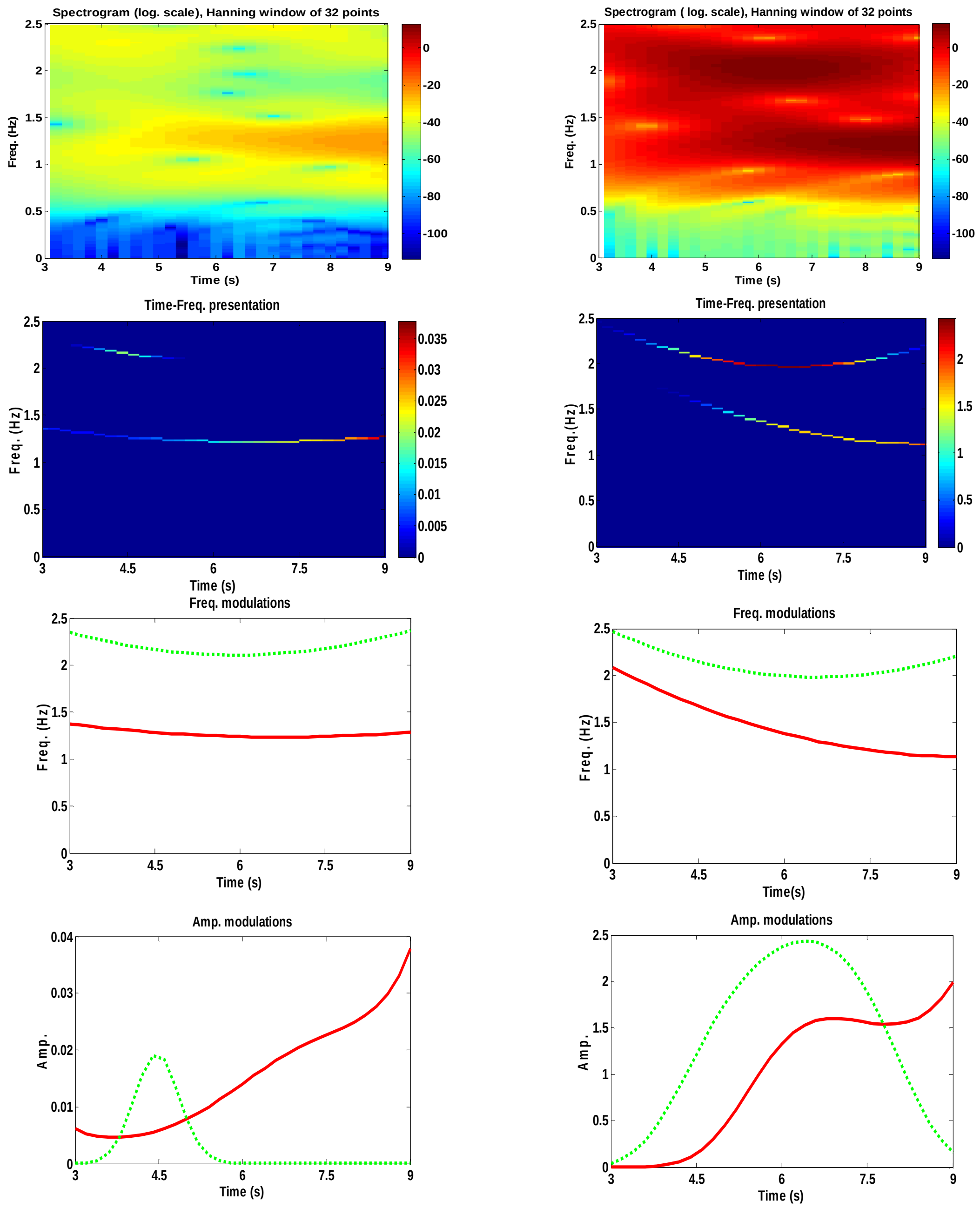
**Figure 8: Analysis of the real-world seismic data recorded at the top of Van-Nuys hotel (1st column left), San-José building (2nd column middle) and Sherman-Oaks building (3rd column right) when using the polynomial damping function model ($\alpha_i$ variable): The spectrograms calculated with Hann window of 15 points (1st line top), the estimated amplitude modulations (2nd line middle), and the estimated frequency modulations (3rd line bottom)**

Fig. 8 presents the results of the polynomial damping function model, when studied over the three real-world seismic data of the buildings presented in section 4. The improved performance could be clearly noticed, the amplitude variations are in agreement with the spectrogram, the frequency variations are acceptable as well, and the normalized error has been decreased noticeably.

It has been shown that the amplitude and frequency content of the seismic data are well extracted when applying the polynomial damping function model; this model has proven to be a good choice when dealing with seismic data.

### *4.2 Analysis using simulated seismic signals*

In this section, the polynomial damping function model is applied over the range 3-9 s of the simulated seismic signal presented in section 2, where the seismic ground excitations correspond to those in Fig. 4.




Normalized Error= 0.0399    Normalized Error= 0.01662

**Figure 9: Analysis of the simulated seismic data recorded at the top of the Sherman-Oaks building when excited by seismic ground excitations (1st column) and the real-world seismic data recorded at the top of Sherman-Oaks building during the Northridge earthquake of Jan. 17, 1994 (2nd column): The spectrogram calculated with Hann window of 32 points (1st line top), the time-frequency representation (2nd line), the estimated frequency modulations (3rd line), and the estimated amplitude modulations (4th line)**

Thanks to Fig. 4 the mode at 0.4 Hz doesn't belong to the Brune model frequency range of $[f_c = 0.6Hz, f_{\max} = 5Hz]$, which justifies why this mode is less influenced by the seism. Accordingly, and in order to highlight the effect of the seism on the simulated signals as well as the real ones, we applied a high-pass filter with $f_c = 0.6Hz$ to eliminate the component at $0.4Hz$.

From Fig.9, we can conclude that the two components and their variations are correctly identified for both cases when applying the polynomial damping function model. Meanwhile, this model performs better with the simulated seismic signal than with the real-world seismic one, indeed the normalized root mean square error of the simulated signal is about two times smaller than the seismic data.

The results of the real-world seismic data could be justified because for such data the modal parameters (such as natural frequency, damping ratio, and mode shapes) are always time-variant, however, for the simulated seismic signal the modal parameters are constant and are a priori known. Nevertheless, the polynomial damping function model has proven its efficiency even when dealing with the physical modeling of the building motion.

## 5. Conclusions

Unlike input-output techniques for which building response is evaluated by knowing both the input signal (for example, seismic ground motion) and the output signal corresponding to building motion (for example, motions recorded on the roof), we propose in this paper an Output-Only Modal Analysis method, considering the input as unknown. The model proposed in this paper is the polynomial damping function model based on the time-variant damping coefficient and applied in the context of seismic vibrations, which are characterized by non-linearity, non-stationarity and short-durations.

The proposed model has led to significantly improved estimations of both the amplitude and the frequency modulations as compared to the damped amplitude & polynomial frequency model, the variations of the multi-component seismic signals were tracked correctly over the very short duration of 6 seconds, and the normalized error for each of the studied segments for the concerned signals was noticeably low. Furthermore, this model is more adapted to the civil engineering buildings that are characterized by exponential damping functions, and it is in well accordance with the physical model of the building motion.

In future we intend to study the applicability of this Output-Only signal model in extracting the dynamic properties of the buildings, such as the natural frequency and the damping ratios.

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